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Evaluation of the Wave Characteristics at the Mediterranean Coast of Israel

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Abstract

Different methods of evaluation of long-term and extreme wave conditions were tested using wave data gathered over an 18 yr period at Ashdod, on the Mediterranean coast of Israel. Long-term joint density probability of wave heights and periods estimated for Ashdod is used to evaluate confidence domains for different confidence coefficients. Extreme values are shown to be conservatively estimated by a Gumbell distribution, according to which the 100 yr average-recurrence deep-water significant wave height is about 8.70 m.

Nomenclature

A, a_0, a_1	— constants
B, b	— constants
C_1, C_2, c	— constants
e, exp	— 2.71828...
H	— variable
H, H_s	— variables
H', H'_i	— i-th value in the sample of deep-water significant wave heights
H_c	— scale factor (constant)
H_0	— lower limit of H_s variable (Weibull distribution)
H_1	— upper limiting bound of the variable H_s (Asymptote III)
H^*	— modal value of the log-normal density
$\bar{H}$	— mean value of the sample of H_s values
i	— index, indicates rank of a variable in a sample of values
K	— correlation coefficient between data in a sample (Gumbel, Asymptote I)
L	— economic lifetime of a structure
ln	— natural logarithm
n	— number of trials
$n(x_i)$	— number of trials at level x_i of a continuous process variate x
$n(H_s)$	— average number of chances to occur per year which has a sea state H_s

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$P(X \leq x)$ — cumulative probability of occurrence of X values smaller than x

$P(x \leq x_i)$ — cumulative probability of non-exceedance of x_i value in 1 trial

$P(H_s \leq H_s^i)$ — cumulative probability of occurrence of deep-water significant wave heights smaller than H_s^i

$p(H_s^i)$ — probability density function of H_s^i

$p(T^i)$ — probability density function of T^i

$p(H_s, T)$, $p(H_s^i, T^i)$ — joint probability density function of H_s and T values

$p(r, \theta)$ — joint probability density of r and θ values

$q(x_i)$ — power of the exponent of the asymptotic expression of the cumulative distribution function (see Ochi, 1978a, p. 65)

R — return period

r — risk

r — random variable, radial coordinate

r_i — radial coordinate of a point in the r, θ plane

S_H — standard deviation of the sample of H_s values

S_T — standard deviation of the sample of T values

T — variable in the sample of wave periods corresponding to the sample of H_s values

T^i — variable wave period corresponding to the H_s^i value

$\bar{T}$ — mean value of the sample of T values

T^* — modal value of the log-normal probability density distribution function of the T values

X — $\ln H_s$ value of a variable in a sample

$\bar{X}$ — mean value of the natural logarithms of the H_s values, $\overline{\ln H_s}$

Y — $\ln T$

$\bar{Y}$ — $\overline{\ln T}$, similar to $\bar{X}$

y — reduced variate in the Gumbel method

γ_i — constant

γ — confidence coefficient

$\phi(H_s \leq H_s^i)$ — extreme cumulative probability function of H_s

λ — level (value) of the joint probability density function

Π — 3.14159...

$\rho_{H,T}$ — correlation coefficient of H_s and T values in a sample of wave data

ρ_{XY} — correlation coefficients of X, Y values

σ_x — standard deviation of the population of $\ln H$ values

σ_y — standard deviation of the population of $\ln T$ values

θ — random variable, angular coordinate

1. Introduction

The present study describes the evaluation of wave characteristics on the Mediterranean coast of Israel and tests different methods of analysis of existing wave data. The basic set of wave data used here were collected at Ashdod from 1958 by the Coast Study Division of the Israel Ports Authority. This is the largest and most extensive set of wave data gathered to date on the Mediterranean coast of Israel. Hence, this set of data has been used as an almost unique data basis for the design of coastal and marine structures in Israel.

The safety of any coastal or marine structure, as well as its optimal utilization, primarily depend on the nature and reliability of the data used for its design. Of particular importance is information on wave characteristics at the site as its analysis allows:

a) the evaluation of the combined and marginal probability of occurrence of wave heights, periods and directions, which are used in the optimization of the functions of a structure (e.g., yearly average number of days which a ship can safely moor or unload at a certain berth), and

b) the evaluation of the probability of occurrence of extreme sea states, which is required for structural design, i.e. for the determination of the safety level of the structure.

The wave climate at a site is determined by monitoring the waves there for a long period. At present, the monitoring usually consists of wave measurements performed using sophisticated instruments. The "instrumental measurements" consist of 10- to 20-min intervals taken every 3 or 6 h or only daily, and they usually extend over a period of, at most, a few years. In the past, when sophisticated wave measuring instruments were not available, the monitoring was carried out using simple methods. The simplest one consisted of "visual observations" and the quality of the data obtained depended on the skills of the observers and their experience. A more complex method consisted of "visual wave measurements", by which the wave parameters were visually determined by means of a graduated stick, binoculars and a stopwatch. At Ashdod, the "visual wave measurement" method was employed until 1975 and the "instrumental wave measurement" method from 1977. As visual monitoring could be performed only during daylight, the "visual measurements" were performed less frequently than the "instrumental measurements" (only once to three times per day). Although such data seem to be very sparse, the reliable evaluation of the long-term combined and marginal probability distributions of the wave heights, periods and directions is feasible because of the large number of data.

The reliable evaluation of extreme sea states (i.e. significant wave heights with large return periods) is also feasible because it covers relatively many years and a reasonably large sample of yearly sea-state maxima is available.

A previous study conducted by Rosen and Vajda (1978) indicated that the wave climate in deep water at Ashdod fairly well represents the deep-water wave climate on the Mediterranean coast of Israel. That study was based on "visual wave measurements" performed simultaneously for 25 months at Ashdod (southern section of the coast) and at Hadera (about 100 km north of Ashdod). Therefore, to evaluate the wave characteristics on the Mediterranean coast of Israel, the raw data gathered at Ashdod were initially processed in such a way as to form two homogeneous samples of data, one of daily maximum deep-water significant wave heights, the other of yearly maximum deep-water significant wave heights. Further, the Ashdod wave data are compared here with wave data gathered and published by the U.S. Army Naval Weather Service Command (1970).

In the present study the significant wave height H_s refers to the deep-water significant wave height unless otherwise specified.

2. Description of the Sets of Wave Data Used in the Present Study

Raw Data

The raw data available for the present study can be divided into four groups. These groups were derived according to the methods of measurement and the periods of data acquisition.

Group I (1958–1971). Three daily visual wave measurements were carried out usually at 0600,0900 and 1200 GMT. The visually measured wave height represented the height of the highest breaking wave during 5 to 10 min of continuous monitoring of the elevations of the breakers' crests above MSL datum. These elevations were continuously determined by a shore-based observer, using a graduated stick, who aligned each breaker crest with the horizon (see Fig. 1). The estimated height of the crest was later corrected by removing the influence of the tide on the sea level. The tide was recorded at Ashdod hourly, using a mareograph which was installed, for the period 1958–1965, inside the cooling basin of the Eshkol electric power station and, since 1966, inside Ashdod Port. Further, the maximum breaker height was determined by assuming the highest crest to be 75% of the total breaker height. The use of this assumption, made by the Coast Study Division of the Israel Ports Authority, leads to somewhat conservative values (higher waves). This has been indicated by published data. (Coastal Engineering Research Center, 1977, Vol. II, fig. 7–45, p. 7–82; Vol. III, fig. C-6, p. C-35).

The corresponding main wave direction at a water depth of about 30 m was visually determined from a high point on the shore using binoculars and a compass. The wave period was obtained from visual observation by timing 10 consecutive waves (11 well-formed crests) as they passed a pole located about 1 km offshore. Assuming that the smaller waves were probably not counted, the visual period is assumed to be closer to the significant period, and is considered as such in the following text. This assumption was supported also by the hindcasted values for one of the most severe storms (Stiassnie, 1978).

Group II (1973–1975). Waves were visually measured at the same times as in Group I, but their heights were determined relative to a pole placed at 12 m water depth. Wave directions and periods were measured in the same way as for Group I. It must be stressed that these two bodies of data are incomplete since consecutive observations from one or more

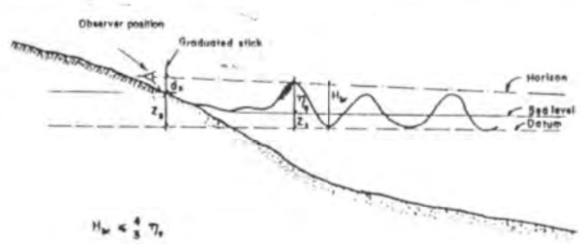

Fig. 1. Sketch of wave measurement method.

days are often missing. Moreover, the three daily maximum measurements cover only a quarter of a day.

Group III (1977, May 1978–March 1979). The wave data included in this group refer to instrumental wave measurements performed offshore from Ashdod, at a depth of approximately 19 m using a Datawell waverider. The waves were measured continuously for 10 min at 3-h intervals and were recorded on paper only. Evaluation of the significant wave heights from these paper records was not performed until recently. In order to preserve homogeneity of the sample of wave data used, the data of Group III were, in general, not included in the statistical evaluation. However, the wave records of yearly maxima were processed manually. The significant wave heights evaluated from them allowed us to increase the size of the sample of yearly maxima of significant wave heights. The wave periods were determined also from the paper records, while the wave directions were obtained, for this Group, at the same time and in the same way as in Groups I and II.

Group IV. The wave data included in this group refer to wave measurements performed with the same wave rider buoy as in Group III but the data were recorded on magnetic tape and processed on a mini computer (Nova 3) at the Israel Coastal and Marine Engineering Research Institute, Technion – Israel Institute of Technology, Haifa, Israel. The data consists of measurements performed at Hadera during January–April 1978 and at Ashdod from April 1979. The waves were measured also at 3-h intervals as in Group III, but for a duration of about 20 min. These measurements were used only to obtain the yearly maxima of significant wave heights in order to have homogeneity of the data, as was explained previously. The wave direction was determined at the same hours and in the same manner as in Groups I, II and III.

Processing of the Raw Wave Data

The raw wave data used were processed in such a way as to obtain a homogeneous sample of deep-water significant waves. Therefore, the following constraints were introduced: (a) only the maximum daily wave height was selected from the three daily visual measurements available in Groups I and II, together with its respective period and direction, and (b) differences among observers and among estimates of durations was neglected, so as to give each one an equal weight; the duration of an observation was taken as 5 min. The choice of only the maximum daily wave height was meant to limit the dependence on consecutive measurements, as the other two values were obtained in the same quarter of the day.

The evaluation of the deep-water significant values was performed in the following ways:

a) The wave heights of Group I, which consisted, as mentioned in Sect. 2, of maximum breaker heights during 5-min periods, were transformed into deep-water significant wave heights preserving the same significant wave period in the following way.

First, for each maximum breaker height, $H_{br,max}$, the deep-water maximum wave height, $H_{0,max}$, during 5 min (300 s) was determined. Since the breaking depth was unknown, use was made of the empirical formula of Komar and Gaughan (1972), also taking into account the refraction coefficient and the shore slope (which is about 1:80). Then this formula became (in metric units):

$$H_{0,max} = (H_{br,max}^{1.25}/0.53\,T_s^{0.5}) \cdot (\cos\alpha_{br}/\cos\alpha_0)^{1.25} \quad (1)$$

where T_s = significant wave period $\cong T$; T = wave period (in seconds), visually determined by dividing the total time during which a number of waves passed a fixed point to the number of waves (10) counted during this period; α_0 = angle between the deep-water wave crest and the shore line (in degrees); α_b = angle between the breaker line and the shore line.

Secondly, assuming that the short-term distribution of the wave heights behaves according to the Rayleigh distribution, the deep-water significant wave heights were calculated. The deep-water maximum wave height obtained was assumed to represent the most probable highest deep-water wave during the 5-min period. Thus, using the relationship between the most probable highest wave and the significant wave height according to Longuet-Higgins (1952, formula 2), the deep-water significant wave height was obtained:

$$H_{0,max} = [\tfrac{1}{2}\ln(N)]^{\frac{1}{2}} \cdot H, \quad (2)$$

where N is the number of waves during the observation period: $N = 300/T$.

Finally, hindcast analysis performed for one of the heaviest recorded storms during the period covered by Group I confirmed, in another way, the deep-water values obtained from the visual wave measurements. The results of the hindcast analysis have shown consistently similar wave values with those obtained from the measured breakers (Stiassnie, 1978).

b) The evaluation of the deep-water significant wave heights of the other groups of data was performed again using the values of the maximum waves, assuming parallel straight contour lines and using refraction and shoaling coefficients presented by Coastal Engineering Research Center (1977, Vol. II, fig. 7–45, p. 7–82; Vol. III, fig. C-6, p. C-35). Afterwards, the transformation from deep-water maximum wave height to deep water significant wave height was performed as for Group I.

Samples of Wave Data

The data processed, as explained above, were divided in two wave samples: the first one, with a relatively large number (5168) of daily maxima of deep-water significant waves, the second with relatively few values (18) containing yearly maxima of deep-water significant wave heights and associated periods. A third sample of wave data acquired by the U.S. Army Naval Weather Service Command (1970) in the eastern Mediterranean by visual observation from ships was used for comparison with the former one. These samples formed the basis for the statistical analyses performed in the present study.

3. Evaluation of the Marginal Probability Distribution of Significant Wave Heights and Wave Periods

In order to evaluate the long-term cumulative probability distribution of the significant wave heights at Ashdod, use was made of a relatively large sample (5168 values) of daily maxima of deep-water significant wave heights. Different authors used different distributions to describe the cumulative probability distribution of significant wave heights and periods. Such distributions include the exponential, Weibull, normal and log-normal distributions. These are the most commonly used ones and are presented below.

Exponential Distribution

This distribution has been used and advocated by many authors for the evaluation of the long-term cumulative probability distribution of significant wave heights. Copeiro (1978) reaches the conclusion that: "the exponential function could then describe statistically the growth of the sea caused by wind fields reaching the observation site, originated by differential pressure centers".

The technique is to plot $\log(1 - P(H_s \leqslant H_s^i))^{-1}$ versus H_s^i through the higher values of H_s^i, assuming them to be exponentially distributed. Here: $P(H_s \leqslant H_s^i)$ = cumulative probability of occurrence of deep-water significant wave heights smaller than H_s^i. H_s^i is the i-th value in a sample of deep-water significant wave heights. Accordingly, the sample of data was plotted on semi-log paper in Fig. 2. Following Bretschneider and Rocheleau (1978), a straight line was drawn through the higher H_s values, using least-squares regression, in order to obtain the exponential distribution of the deep-water significant wave heights.

The function obtained may be expressed as:

$$P(H_s \leqq H_s^i) = 1 - 12.08[10^{-0.68H_s^i}] \quad (1)$$

$$P(H_s \leqq H_s^i) = 1 - 12.08\exp[-1.565H_s^i] \quad (2)$$

Weibull Distribution

This distribution, proposed originally by Weibull, was suggested for application to coastal engineering by Bretschneider (1964). In fact it is the general case

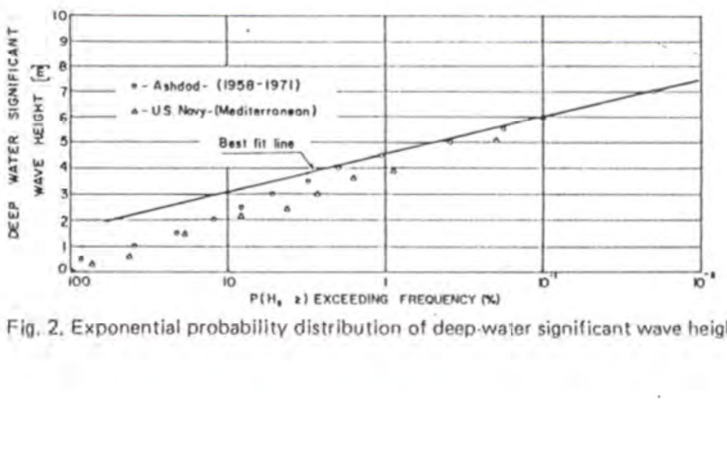

Fig. 2. Exponential probability distribution of deep-water significant wave heights.

of the exponential distribution, the exponent being raised to some power, while in the exponential distribution the power of the exponent is 1.

The general form of the Weibull distribution is given by:

$$P(X \leq x) = 1 - \exp(-(x-a)/b)^c \tag{3}$$

For the distribution of significant waves the following form is generally used:

$$P(H_s \leq H_s^i) = 1 - \exp[-((H_s^i - H_0)/(H_c - H_0))^\gamma] \tag{4}$$

where a, H_0 = the lower limit of the variable, b, H_c = the scale factor and c, γ = the shape factor.

If $a = 0$ then the distribution is known as the Frechet distribution. The use of this distribution has been advocated by many authors (Battjes, 1972; Houmb & Overvik, 1977), the most prominent being the International Commission for the Reception of Large Ships (1979). When the data are plotted on Weibull paper it is expected that they will fit a straight line. However, to do so, a good choice of the H_0, i.e. the lower limit of the variable, has to be made. This value of H_s is found when $1 - P(H_s \leq H_s^i) = 1/e$ and the straight line is plotted using the least-squares method.

Ochi (1978a) showed that the Weibull distribution of the significant wave height does not fit well the data for small significant wave heights but fits reasonably well for large significant wave heights. In order to test whether these findings also apply to our sample, the data were plotted on Weibull paper using these techniques to find the cumulative distribution. These are presented in Fig. 3. As recommended by Houmb (1976) and by the International Commission for the Reception of Large Ships (1979) the line drawn was fitted to the large wave heights ($H_s \geq 4$ m) using the least-squares method. From it the cumulative distribution of deep-water significant waves (assuming them to be Weibull distributed) was found to be given by the expression:

$$P(H_s \leq H_s^i) = 1 - \exp[-(H_s^i/1.33)^{1.29}]$$
$$= 1 - \exp[-0.692(H_s^i)^{1.29}] \tag{5}$$

The Weibull probability density distribution for this expression was plotted in Fig. 6 together with the histogram and other distributions.

By looking at Fig. 3 it can be seen that the sample of data gathered by the U.S. Navy in the eastern Mediterranean falls consistently below the Ashdod data with the exception of one extreme value. However, the plotting alignment is parallel to that of

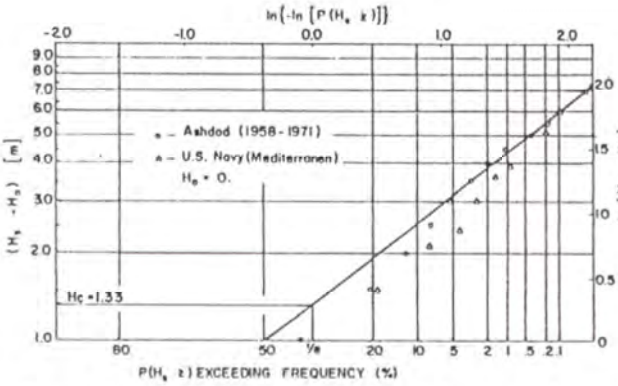

Fig. 3. Weibull probability distribution of deep-water significant wave heights.

Ashdod data. An explanation for these results may be that the visual wave measurements tend to exaggerate the wave heights while, according to Jardine (1979), visual ship observations of significant wave heights are very close to instrumentally measured significant wave heights.

Normal Distribution

The use of this method is possible if the significant wave heights are assumed to follow a Gaussian distribution. By evaluating the mean and standard deviation of the deep-water significant wave heights, the following expression was obtained:

$$P(H_s \leq H_s^i) \tag{6}$$
$$= \frac{1}{0.88 \cdot \sqrt{2\pi}} \int_{-\infty}^{H_s^i} \exp[-\tfrac{1}{2}((H - 1.15)/0.88)^2]dH$$

The resulting distribution is plotted, together with the actual sample data, in Fig. 4 showing relatively poor agreement between the two. A straight line may be fitted through some of the data (the higher wave

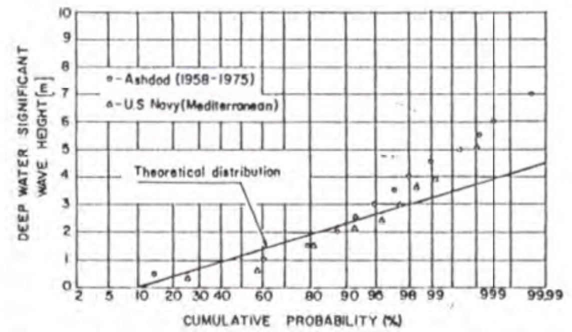

Fig. 4. Normal cumulative probability distribution of deep-water significant wave heights.

heights). Yet, even so, the distribution is not fitted for values of $P(H_s^l)$ smaller than 0.8. Also, because the cumulative normal probability function ranges from $-\infty$ to ∞, the region below zero has no physical meaning for wave heights or periods.

Log-Normal Distribution

Darbyshire (1961) and Draper (1963) concluded that the long-term distribution of wave heights and wave periods can best be estimated by assuming a Gaussian distribution of the natural logarithm of the wave heights or periods and that this distribution applies to any wave height (period) parameter (average, significant, etc.). Ochi (1978a) concluded that this distribution fits the histogram of significant wave heights and periods reasonably well, but fails to suit the data for cumulative probabilities larger than 0.99. In the present study, the sample of daily data was used for the evaluation of the cumulative probability distribution of the deep-water significant wave heights. The following expression was obtained:

$$P(H_s \leqq H_s') = \frac{1}{0.68\sqrt{2\pi}} \int_0^{H_s'} \frac{1}{H} \cdot \exp[-\tfrac{1}{2}((\ln H + 0.093)/0.68)^2]\, dH \quad (7)$$

This log-normal distribution was obtained from the mean and standard deviation of the deep-water significant wave heights sample, using the method indicated by Weiss (1957) by which:

$$\sigma_x = \sqrt{\ln\left[1 + \left(\frac{S_H}{\bar{H}}\right)^2\right]} \quad \text{and} \quad \bar{X} = \ln \bar{H} - \tfrac{1}{2}\sigma_x^2 \quad (8)$$

where $\bar{X} = \overline{\ln H}$, $H = H_s$ = deep-water significant wave height, S_H = standard deviation of the H sample, $\bar{H}$ = mean value of H, $\bar{X}$ = mean value of the natural logarithms of H, and σ_x = standard deviation of the $\ln(H)$ values population. The resulting distribution was plotted together with the actual data of the sample in Fig. 5.

The data fit the theoretical line reasonably well for probability values smaller than 0.98. Beyond that value the fit becomes poor. However, in order to obtain a better picture of the fit of the data to the theoretical distribution obtained, another method of analysis is used. It compares the density probability function and the data histogram. Thus, the log-normal density probability distribution of the deep-water significant wave heights was evaluated to be given by the expression:

$$p(H_s') = (1/H_s' \cdot \sqrt{2\pi}) \exp[-\tfrac{1}{2}((\ln H_s' + 0.093)/0.68)^2] \quad (9)$$

The histogram of the actual data is presented in Fig. 5. Also shown in this figure is the Weibull probability density distribution for comparison with the findings of Ochi (1978a). He found that while comparison between curves of the cumulative distribution function using log-normal and Weibull distributions showed similar goodness of fit, creating the impression of a non-significant difference between them, the

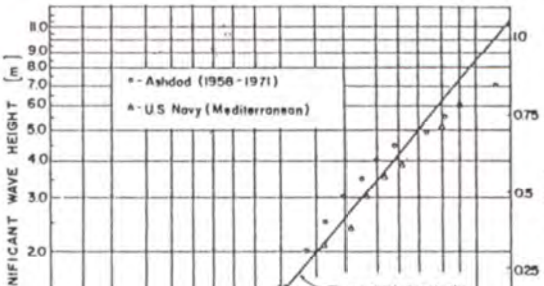

Fig. 5. Log-normal cumulative probability distribution of deep-water significant wave heights.

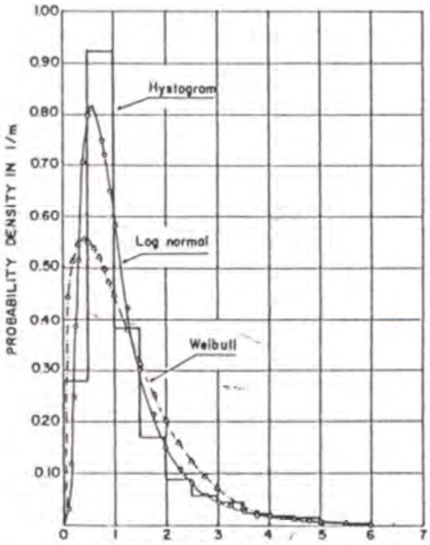

Fig. 6. Deep-water significant wave height offshore of Ashdod: histogram and density probability distributions.

comparison among plots of the histogram and the probability density, Weibull and log-normal functions accentuate the difference between the two distributions. From Fig. 6 it can be seen that the Weibull distribution fits the histogram less well for small significant wave heights, but much better for the large ones. The relatively good fit of the log-normal distribution to the histogram indicates that this (log-normal) distribution can be used to estimate the long-term probability density distribution of significant wave heights, and the Weibull distribution can be used to evaluate extreme values.

Marginal Probability Distribution of the Wave Periods

The sample of visual wave periods (corresponding to the maximum daily significant wave heights) was used to evaluate the log-normal probability density function using Eq. (8). It was based on the work of Ochi (1978a) who obtained a good fit also between the long-term histogram of the wave periods and the long-term log-normal probability density of the wave periods. The resulting expression is given by:

$$p(T^i) = 1/(T^i\sqrt{2\pi})\exp[-\tfrac{1}{2}((\ln T^i - 1.67)/0.305)^2] \quad (10)$$

where $\bar{Y} = \ln T^i$, T^i is the visually determined wave period corresponding to H_s^i, $\bar{Y} = 1.67$, and $\sigma_y = 0.305$.

Equation (10) was plotted together with the histogram in Fig. 7. The comparison shows a good fit of the log-normal distribution with the histogram. This indicates that a log-normal distribution may be used to describe the long-term probability density distribution of the wave periods offshore of Ashdod.

4. Joint Probability Distribution of Deep-Water Significant Wave Heights and Wave Periods, Confidence Domains and Confidence Coefficients

As mentioned before, Ochi (1978a) found that log-normal probability density functions describe the histograms of the significant wave heights and wave periods reasonably well. Hence, he suggested expressing the joint density probability function of the significant wave heights and wave periods in the following manner:

$$p(H_s, T) = \frac{1}{H_s \cdot T \cdot \sigma_x \sigma_y 2\pi\sqrt{1-\rho_{XY}^2}} \cdot \exp\{-(1/2)(1-\rho_{XY}^2)\cdot[((X-\bar{X})/\sigma_x)^2 - 2\rho_{XY}\cdot((X-\bar{X})/\sigma_x)\cdot((Y-\bar{Y})/\sigma_y) + ((Y-\bar{Y})/\sigma_y)^2]\} \quad (11)$$

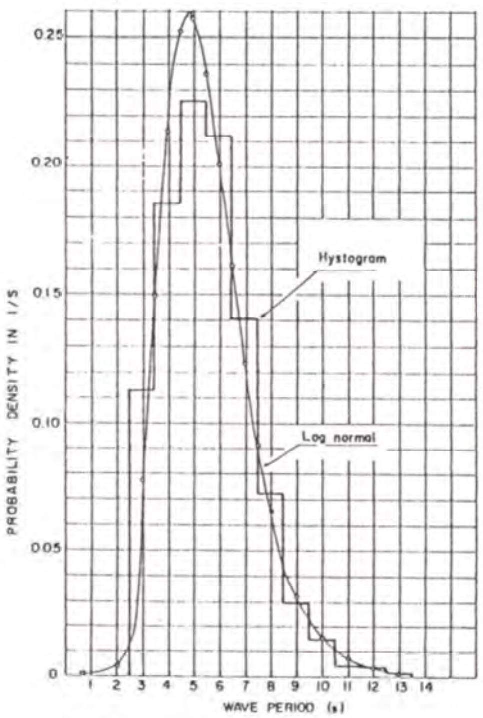

Fig. 7. Wave period at Ashdod, histogram and log-normal density probability distribution.

where $X = \ln H_s$, $Y = \ln T$, ρ_{XY} = correlation coefficient of X, Y values.

$$\rho_{XY} = \frac{\ln(\rho_{HT}^2 \cdot \sqrt{(e^{\sigma_x^2}-1)(e^{\sigma_y^2}-1)}+1)}{\sigma_x \sigma_y} \quad (12)$$

where ρ_{HT} = correlation coefficient of H_s, T values.

Since similar results were found for the Ashdod data as presented above, the same expression was used in this work to describe the long-term joint probability density distribution of wave heights and wave periods. Thus, confidence domains for different confidence coefficients γ were calculated using a numerical integration method and the results were plotted in Fig. 8. The confidence domain is defined here as the area enclosed by the contour curve. This is the intersection of the joint probability density function with a plane parallel to the (H, T) plane at a certain level λ of the joint probability density. The confidence coefficient γ is defined as the cumulative probability of the corresponding domain.

In order to find the confidence domains for different confidence coefficients, the relationship between the confidence coefficient γ and the corresponding values of the joint probability density λ must be found, i.e.

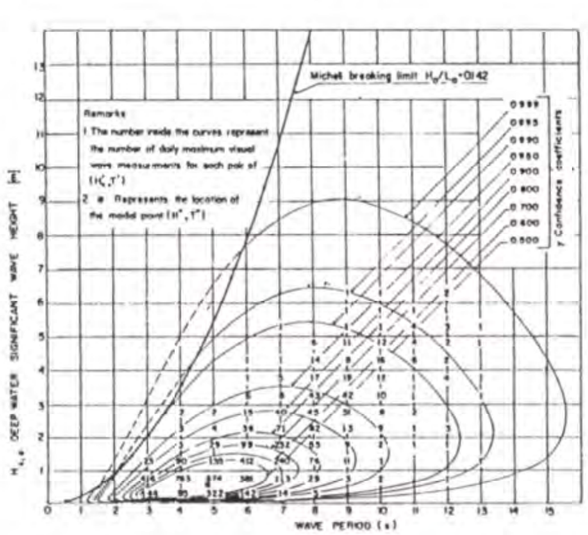

Fig. 8. Domains of significant wave height and period for various confidence coefficients.

$$\gamma = f(\lambda) \quad \text{and} \quad \lambda = p(H_s^i, T^i) \tag{13}$$

According to the definition of γ, the relationship $\gamma = f(\lambda)$ may be calculated by numerical integration of the joint density probability (H_s^i, T^i) on a confidence domain which may be found for different values of λ. After the relationship $\gamma = f(\lambda)$ is calculated, the corresponding values of λ for different values of γ are found by interpolation of the above-mentioned expression. These values of λ allowed us to find the curves enclosing the confidence domains corresponding to different γ values.

Ochi (1978a) used another method, which was based on the assumption that all points on a certain contour, which encloses the confidence domain, may be found by the following expression:

$$\gamma = \int_0^{r_i} f(r \mid \theta) dr \tag{14}$$

where

$$f(r \mid \theta) = \frac{f(r, \theta)}{\int_0^{\infty} f(r, \theta) dr} \tag{15}$$

where the functions $f(r, \theta)$ and $f(r \mid \theta)$ are the joint density probability and the conditional density probability, and r, θ may be found from the expressions:

$$H_s = H^* + r\cos\theta; \quad T = T^* + r\sin\theta \tag{16}$$

where H^*, T^* are the modal values, i.e.

$$H^* = \exp(\bar{X}); \quad T^* = \exp(\bar{Y}) \tag{17}$$

It may be proved theoretically that only in the case when

$$p(r, \theta) = p[r^2 \cdot f_1(\theta) + f_2(\theta)] \tag{18}$$

where $f(\theta)$ is any function of θ, does the contour curve, as defined by Ochi (1978a), become identical with the contour curve of constant joint probability density. However, the function $p(r, \theta)$ cannot be represented in the form of expression (18) in this case because $p(r, \theta)$ is given by the expression:

$$\begin{aligned} p(r, \theta) = {} & \frac{1}{\sigma_x \sigma_y 2\pi \sqrt{1-\rho_{xy}^2}(H^* + \cos\theta)(T^* + r\sin\theta)} \\ & \cdot \exp\left\{ -\frac{1}{2(1-\rho_{xy}^2)} \left[\left(\frac{\ln(H^* + r\cos\theta) - \bar{X}}{\sigma_x} \right)^2 \right.\right. \\ & - 2\rho_{xy} \left(\frac{\ln(H^* + r\cos\theta) - \bar{X}}{\sigma_x} \right) \\ & \cdot \left(\frac{\ln(T^* + r\sin\theta) - \bar{Y}}{\sigma_y} \right) \\ & \left.\left. + \left(\frac{\ln(T^* + r\sin\theta) - \bar{Y}}{\sigma_y} \right)^2 \right] \right\} \end{aligned} \tag{19}$$

In order to show the differences, we also used Ochi's method and we found the contour curves, which are presented in Fig. 9. However, for different points on a certain contour curve, as defined by Ochi, the results obtained give different values of the joint

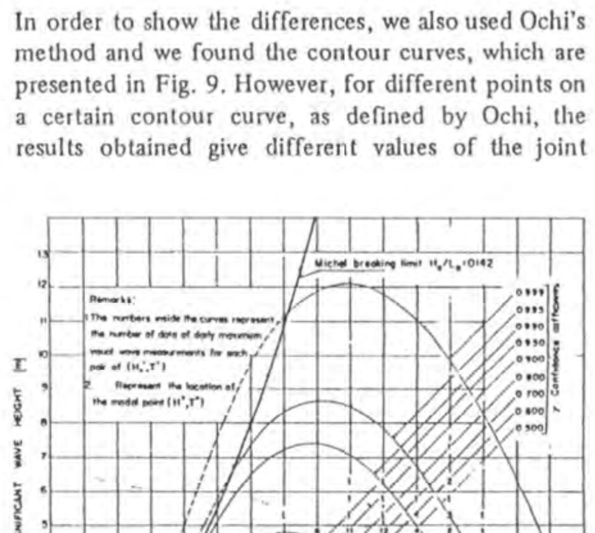

Fig. 9. Domains of significant wave height and period for various confidence coefficients as defined by Ochi (1978a).

probability density. Thus, the area enclosed in a such-defined contour curve does not represent the confidence domain according to its usual definition.

The evaluation of the contour curves was carried up to confidence coefficients of 0.999, although the log-normal probability functions fitted the histograms reasonably well for cumulative probabilities of about 0.99. The evaluation beyond this value was done because it was assumed that the correlation coefficent between the wave heights and periods remains the same even for rare values, and because it may give some estimate of the range of values for such rare conditions.

5. Evaluation of Extreme Sea States

Methods of Evaluation

The data acquired at Ashdod were used for the evaluation of extreme sea states using three approaches.

a) Evaluation on the basis of the yearly maxima of the deep-water significant wave heights using extreme probability functions (Asymptote I, Asymptote III, and log-normal).

b) Evaluation on the basis of the whole set of daily maxima of deep-water significant wave heights and the yearly average number of occurrences of sea states using the Copeiro method.

c) Evaluation on the basis of the whole set of daily maxima of deep-water significant wave heights, assuming the long-term cumulative probability distribution (exponential, Weibull, log-normal) to also describe extreme sea states and assuming that the return period expressed in years can be obtained by transforming it from return period expressed in days.

a) Yearly Maxima

Asymptote I. This distribution expresses the cumulative probability that an extreme sea state H_s will be less than or equal to a given extreme sea state H_s^i by the formula:

$$\Phi(H_s \leq H_s^i) = \exp[-\exp(-y)] \quad (20)$$

where y is termed the reduced variate and is generally given by the expression:

$$y = (H_s^i - a_0)/a_1 \quad (21)$$

where a_0, a_1 are the constants obtained by best fit to the data using linear regression:

$$a_0 = \bar{y} - K a_1 \bar{H}_s\,; \quad a_1 = \sigma_y / \sigma_{H_s} \cdot K \quad (22)$$

where K = correlation coefficient.

When the correlation coefficient K is taken as 1 it is known as the Gumbel fit method. Accordingly, the data of yearly maxima was fitted to an Asymptote I distribution using both the Gumbel solution and least-squares regression to the actual data. The resulting expressions are:

a) fit to actual data:

$$H_s^i = -0.8287[-\ln[\Phi(H_s \leq H_s^i)]] + 4.8876; \quad (23)$$

$$K^2 = 0.9553$$

b) fit by the Gumbel method:

$$H_s^i = -0.8478\ln[-\ln[\Phi(H_s \leq H_s^i)]] + 4.8793; \quad (24)$$

$$K^2 = 1.000$$

These relationships are shown in Fig. 10 and, from the figure, it can be seen that the Gumbel solution gives somewhat larger values than the other curve for large return periods.

Asymptote III. This distribution, originally mentioned by Fisher and Tippet (1928) and later by Gumbel (1958), was also proposed by Borgman (1975) and needs to be given more attention, because it is meant to represent situations in which the variables are limited by some upper boundary. Its expression is given by:

$$\Phi(H_s \leq H_s^i) = \begin{cases} 1 & \text{for } H_s^i > H_1 \\ \exp[-c(H_1 - H_s^i)^\gamma] & \text{for } H_s^i \leq H_1 \end{cases} \quad (25)$$

where H_1 is the upper limiting bound and c, γ are constants.

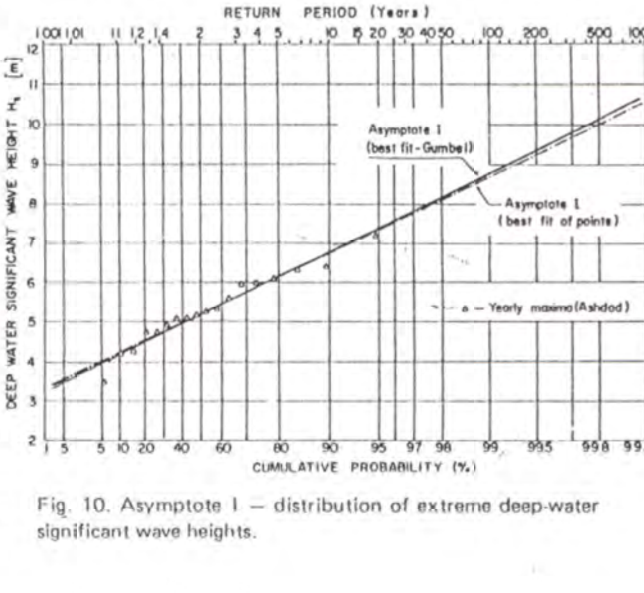

Fig. 10. Asymptote I – distribution of extreme deep-water significant wave heights.

In order to estimate a limiting value of the largest possible deep-water significant wave height at the Mediterranean coast of Israel, use was made of the deep-water wave forecasting curves developed by Bretschneider taking the largest fetch at Ashdod (2300 km) and a wind speed of 100 knots. The resulting value obtained was a wave height of 36 m.

In order to examine the sensitivity of the results for extreme sea states to the limiting values, other limiting values of 15 m, 20 m and 1000 m were also used to find the best-fit line to the data using least-squares regression. The resulting distributions were as follows:

$$\ln(15 - H_s') = 0.0872 \cdot \ln[-\ln[\Phi(H_s \leq H_s')]] + 2.3113 \quad (26)$$

$$\ln(20 - H_s') = 0.057 \cdot \ln[\Phi(H_s \leq H_s')]] + 2.7144 \quad (27)$$

$$\ln(36 - H_s') = 0.0271 \cdot \ln[-\ln[\Phi(H_s \leq H_s')]] + 3.4373 \quad (28)$$

$$\ln(1000 - H_s') = 0.000833 \cdot \ln[-\ln[\Phi(H_s \leq H_s')]] + 6.90286 \quad (29)$$

Since each distribution requires a different plotting scale in order to be plotted on Weibull paper, they were not plotted together on Weibull paper. However, the results for different return periods $[1/\Phi(H_s)]$ are presented in Table 1 and in Fig. 13 together with results for identical Φ using Asymptote I and other methods.

Log-normal. The use of this distribution for the evaluation of extreme values was originally proposed by Weiss (1957). It is presented here for comparative reasons with other distributions. The log-normal distribution obtained (also using Eq. (8)) is shown together with the actual data in Fig. 11 and its expression is given by:

$$\Phi(H_s \leq H_s') = \frac{1}{\sqrt{2\pi} \cdot 0.17} \int_0^{H_s'} \frac{1}{H} \cdot \exp\left[-\frac{1}{2}\left(\frac{\ln H - 1.66}{0.17}\right)^2\right] dH \quad (30)$$

Quite surprisingly, the fit to the actual data is quite good. Values for representative return periods are presented in Table 1 and in Fig. 13 with other distributions.

b) Copeiro Method

Copeiro (1978) proposed a method by which the statistically dependent events of a physical continuous process are transformed in a sample of discrete events with statistical independence, and showed the application of the method for the evaluation of extreme significant wave heights. According to it, each sea state (variate H_s) is assigned an average duration per year and the total average number of occurrences of a sea state (H_s) thus becomes a function of the sea state itself. Therefore – while for a sample of discrete events the probability $\Phi(x \leqslant x_i)$ that an event will not be exceeded in n trials is given by $\Phi(x \leqslant x_i) = [P(x \leqslant x_i)]^n$, where $P(x \leqslant x_i)$ is the probability of non-exceedance in one trial – for a continuous evolution process the number of trials n is a function of x itself, i.e. $n = n(x)$ and then

$$\Phi(x \leq x_i) = [P(x < x_i)]^{n(x)} \quad (31)$$

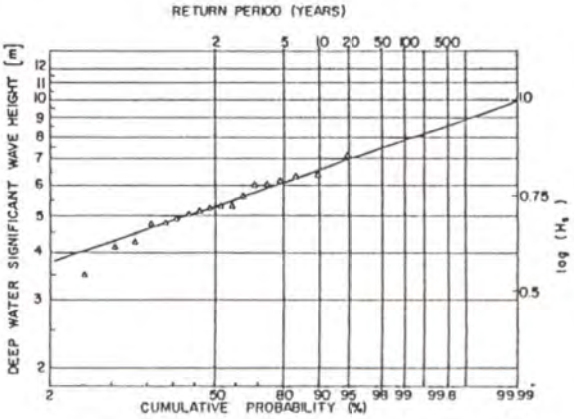

Fig. 11. Log-normal distribution of extreme deep-water significant wave heights.

For significant wave heights, Copeiro found that the $n(H_s)$ function was satisfied by the empirical linear relationship:

$$n(H_s) = A\,(H_s' - B) \quad (H_s', \text{ and } B \text{ are in meters}) \quad (32)$$

Further, according to his findings, the most suitable distribution for $P(H_s \leqslant H_s^i)$ is the exponential distribution, while the Weibull distribution is less suitable and the log-normal is unsuitable due to its poor fit to extreme values. According to the method described above, an evaluation of the $n(H_s)$ function was done on the basis of the available data concerning the yearly average durations of sea states at Ashdod. The data was plotted in Fig. 12 and from it the function obtained, assuming linear dependence, was

$$n(H_s) = 69H_s + 34 \quad (H_s \text{ is in meters}) \quad (33)$$

Afterwards, using the exponential and Weibull distributions previously obtained in Sec. 4, extreme values were evaluated on the basis of the Copeiro method, i.e.

$$\Phi(H_s \leq H_s') = [1 - \exp(-0.692H_s')^{1.29}]^{(69H_s'+34)} \quad (34)$$

using the Weibull function and

$$\Phi(H_s \leq H_s') = [1 - 12.08\exp(-1.565H_s')]^{(69H_s'+34)} \quad (35)$$

using the exponential function.

It should be mentioned here that the return period is already expressed in years and not in days, due to the way $n(H_s)$ was obtained.

The results are presented in Table 1 for comparison with the other methods of evaluation of extreme values. They are also presented in Fig. 13.

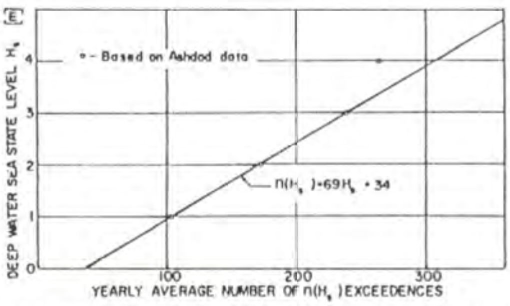

Fig. 12. Yearly average number of exceedences of sea state $n(H_S)$.

TABLE 1. Extreme Values of Deep-Water Significant Wave Heights Offshore of Ashdod as a Function of Return Period and Method of Evaluation

Average return period (years)	Sample of yearly maxima (Use of extreme probability functions)						
	Asymptote I		Asymptote III				Log-normal
	Gumbell method	Best fit to points	$H_L=15_m$	$H_L=20_m$	$H_L=36_m$	$H_L=1000_m$	
2	5.15	5.19	5.23	5.22	5.20	5.19	5.24
5	6.15	6.15	6.15	6.14	6.14	6.15	6.05
10	6.79	6.75	6.71	6.72	6.74	6.75	6.52
20	7.40	7.35	7.21	7.26	7.30	7.34	6.94
50	8.19	8.12	7.82	7.91	8.02	8.11	7.44
100	8.78	8.70	8.25	8.39	8.54	8.69	7.80
200	9.37	9.28	8.64	8.84	9.06	9.28	8.14
400	9.96	9.85	9.02	9.27	9.56	9.84	8.46
500	10.15	10.04	9.13	9.41	9.72	10.02	8.57
1000	10.74	10.61	9.48	9.82	10.21	10.59	8.88

Average return period (years)	Sample of data obtained at short time intervals (Use of long term probability functions)					
	Extrapolation transforming the return period in days				Copeiro method	
	Normal distr.	Log normal distr.	Exponential distr.	Weibull distr.	Exponential distr.	Weibull distr.
2	3.79	6.98	5.80	5.74	5.70	5.82
5	4.03	8.40	6.39	6.35	6.50	6.45
10	4.20	9.57	6.83	6.80	7.02	6.99
20	4.36		7.27	7.24	7.52	7.48
50	4.56		7.86	7.81	8.17	8.11
100	4.71	14.19	8.30	8.23	8.65	8.56
200			8.74	8.65	9.12	9.00
400			9.19	9.07	9.60	9.43
500			9.33	9.20	9.76	9.57
1000			9.77	9.60	10.22	10.00

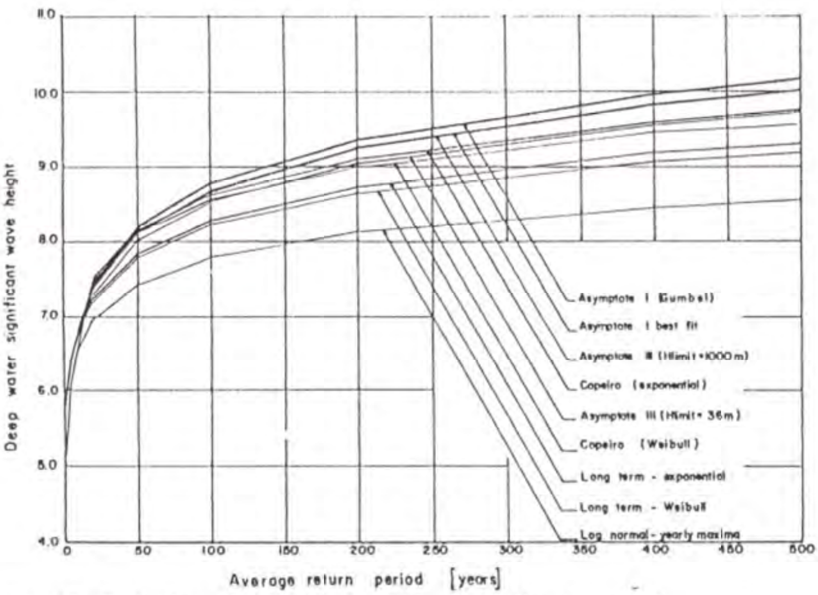

Fig. 13. Evaluation of extreme sea states by different methods.

c) The Long-Term Cumulative Probability Distribution

As explained before, the usual size of the sample of data of yearly maxima is relatively too small to allow a reliable estimate of extreme values (with large return periods). It was also explained why a sample of data taken at constant short time intervals prevents its use for the estimate of extreme values. On the other hand, because the latter sample contains a large quantity of data, it seemed of interest to compare the extreme values obtained by the other methods with the values obtained on the basis of the whole sample of data taken at short time intervals using the long-term cumulative probability distribution. This was done by transforming the average return period expressed originally as a number of short time intervals in two years, by dividing the number of constant time intervals included in a year.

Thus, the long-term cumulative probability functions obtained in Sec. 3, assuming exponential, Weibull, normal and log-normal distributions, were used to evaluate extreme values. The results are presented in Table 1 and in Fig. 13 for comparison with the other methods.

6. Discussion

The wave data gathered at Ashdod were used to evaluate the wave characteristics along the Mediterranean coast of Israel. The analysis did not include wave directions, but was aimed at evaluating the long-term joint and marginal cumulative probabilities of the deep-water significant wave heights and visually observed wave periods, on the one hand, and the evaluation of extreme deep-water significant wave heights, on the other hand.

According to a comparison between the values of significant waves evaluated from usual wave measurements and hindcasted significant wave heights for a major storm, it was concluded in a previous study by Stiassnie (1978) that the wave data evaluated from visual wave measurements, as used here, were sufficiently reliable. Regarding the wave periods, his study indicated that the visual wave periods were close to the significant wave periods, although the study covered only one storm with mostly high waves (>2 m). For smaller waves it is not clear if this applies as well, but using the same assumption conservative values are obtained.

The data used in the present study were composed of the whole sample of visual wave measurements taken at constant short time intervals (daily maxima in our case). The plots of the data sample on Weibull, semilog (exponential), normal and log-normal paper did not permit fitting a straight line through all the data. A reasonably well-fitted straight line could be drawn through the data on wave heights larger than 4 m when Weibull or exponential distributions were assumed, using the method of least-squares regression. The plots of the distribution using normal and log-normal functions do not fit well the data for larger wave heights (which seem to lie on a straight line) because they were built using all the data.

The evaluation of extreme deep-water significant wave heights was tested by employing three methods of calculation. The use of the sample of yearly maxima of deep-water significant wave heights which are assumed independent is somewhat handicapped by the relatively small size of the sample. As pointed out by Borgman (1975), extrapolations beyond twice the sample size are in fact of low reliability. The use of Asymptote III seems more reasonable for the description of a physical process which may be assumed to be limited by an upper bound. As seen from Table 1 and Fig. 13, the results for different limiting values of the upper wave-height bound indicate clearly that the Asymptote I is in fact the limiting case of Asymptote III.

The average return period R (years) represents the average time interval between the occurrences of deep-water wave heights larger than H_s. When designing a structure to last a lifetime L (years) we must be concerned with the occurrence of wave heights, which have only a small chance of occurrence, defined as a small risk value r of being exceeded during the lifetime L of the structure. Thus, the extreme wave height to be used for the design of a structure is given by:

$$R\,(\text{years}) = \frac{1}{1-(1-r)^{1/L}} \tag{36}$$

For a more comprehensive explanation the reader is referred to an article by Borgman (1963).

7. Conclusions

It was found in regard to the long-term wave characteristics in deep water at the Mediterranean coast of Israel that the long-term joint and marginal density probability functions of the significant wave heights and associated periods can be reasonably evaluated assuming log-normal distributions. However, rare values of significant wave height are estimated better assuming a Weibull distribution.

For the evaluation of extreme sea states, the relatively simple method proposed by Gumbel for the use of Asymptote I gives it an advantage over the other ones. Furthermore, the Gumbel method gives the largest values and in fact is the upper bound for the extreme-values distributions. However, its use is possible only for samples with sizes larger than 10. When such samples are not available, and the wave data cover only a few years, the method proposed by Copeiro, using the Weibull probability function, seems to give good estimates of extreme wave heights. The confidence domains of joint probability of occurrence of deep-water significant wave heights and periods can be used to find the critical wave period values associated with a given design significant wave height and confined by a certain confidence interval.

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